Systems of Hamilton-Jacobi equations and differential games

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We consider a non-cooperative differential game for two players, with infinite horizon and exponentially discounted cost. If a Nash equilibrium solution exists in feedback form, under suitable regularity conditions the value functions satisfy a system of Hamilton-Jacobi equations. In general, however, this system is highly nonlinear and difficult to study.

The talk will describe a new analytical approach, based on a homotopy method. The original problem is embedded in a family of problems depending on an auxiliary parameter, accounting for the strength" of the second player. When the parameter is zero, the second player has no power to influence the evolution of the system. He thus adopts a myopic strategy, and the differential game is reduced to an optimal control problem for the first player.

As the parameter becomes strictly positive, one has a genuine differential game. Information on the existence, uniqueness or multiplicity of solutions can be obtained by studying a particular bifurcation problem. Examples show that the approach is also naturally motivated by some economic models.